

NGR-52-012-002

February 1967

TECHNION RESEARCH & DEVELOPMENT

FOUNDATION, HAIFA, ISRAEL

INTERIM REPORT

CREEP OF COMMERCIALY PURE ALUMINUM IN THE LOW
STRAIN REGION UNDER SLIGHTLY VARYING STRESSES

Avraham Berkovits

Technion - Israel Institute of Technology
Department of Aeronautical Engineering
Haifa, Israel

TAE REPORT No. 63

N67-84169

FACILITY FORM 602

(ACCESSION NUMBER)

52

(PAGES)

CR-84785

(NASA CR OR TMX OR AD NUMBER)

(THRU)

none

(CODE)

(CATEGORY)

NGR-52-012-002

February 1967

INTERIM REPORT

CREEP OF COMMERCIALY PURE ALUMINUM IN THE LOW
STRAIN REGION UNDER SLIGHTLY VARYING STRESSES

Avraham Berkovits

Technion-Israel Institute of Technology
Department of Aeronautical Engineering
Haifa, Israel.

TAE REPORT No. 63

The research reported in this document has been sponsored
by the NATIONAL AERONAUTICS AND SPACE ADMINISTRATION,
Washington, D.C., under Grant NGR-52-012-002

SUMMARY

An experimental and analytical investigation currently in progress, on the creep of metals in the primary, low strain region after an incremental change in stress occurs, is described. Metals being studied are commercially pure aluminum sheet and AU-4G1 aluminum-alloy sheet. Details of the experimental set-up and procedures are presented, as well as methods used in data reduction. Preliminary results of creep tests conducted on commercially pure aluminum at 200°C (392°F) indicate agreement with a linear viscoelastic analysis proposed in the literature. The direction of future work in the research program is indicated.

CONTENTS

Summary	i
List of Figures	iii
List of Symbols	iv
Introduction	i
Test Specimens	3
Test Equipment and Instrumentation	4
Test Procedure	13
Linear Equation for Creep after Incremental Change in Stress	15
Methods of Analysis of Experimental Data	17
Preliminary Experimental Results	25
Concluding Remarks	27
References	28
Acknowledgement	30

LIST OF FIGURES

- Fig. 1. Creep Test Specimen.
- Fig. 2. General View of Experimental Set-up.
- Fig. 3. Lateral Loading Assembly.
- Fig. 4. Circuit for Triggering Oscilloscope and Disconnecting Vibrator from Specimen.
- Fig. 5. 1250 Kg. and 125 Kg. Capacity Load Cells.
- Fig. 6. Instrumented Test Specimen in Tension Grips.
- Fig. 7. Strain-Gage Bridge Circuit.
- Fig. 8. Effect of Incremental Change in Stress on Creep Behavior.
- Fig. 9. Creep Curve for Specimen A57 Resulting from Curve-Fitting Procedure.
- Fig. 10. Beam on Fixed Supports and Subjected to Concentrated Bending and Tensile Forces.
- Fig. 11. Tensile Stress-Strain Curves for Annealed Commercially Pure Aluminum.
- Fig. 12. Creep of Commercially Pure Aluminum Sheet at 200°C after Incremental Change in Tensile Stress.
- Fig. 13. Creep of Commercially Pure Aluminum Sheet at 200°C after Decremental Change in Tensile Stress.
- Fig. 14. Creep of Commercially Pure Aluminum Sheet at 200°C after Decremental Stress Change, Excluding Recovery Time.
- Fig. 15. Relation Between Decrement of Stress and Recovery Time for Commercially Pure Aluminum Sheet at 200°C, $t_0 = 5$ hr.

LIST OF SYMBOLS

A	amplitude of vibration
A_o	amplitude at resonance
a, b	creep constants in Eq. (4)
C	beam end-fixity coefficient
E	modulus of elasticity
E_{creep}	modulus for increasing stress during creep
E_d	dynamic modulus
E_R	reduced modulus
F_{ji}	creep strain parameter, defined by Eq. (10)
f_n	resonant frequency
g	acceleration of gravity
H	axial force
h	sheet thickness
h_o	initial sheet-thickness
I	moment of inertia of specimen cross-section
k_o, k_1, k_2, k_3	creep constants in Eq. (5)
l	gage length of specimen
l_o	initial gage length
M	bending moment
n	cycles of vibration between amplitude measurements
P, Q	linear differential operators
p_r, q_r	creep constants in Eq. (1)
ΔT	difference between test temperature and room temperature

t	time
t_o	time at stress change
W	lateral (bending) force
w	specific weight
x, y	cartesian coordinates
α	coefficient of linear expansion
δ	logarithmic decrement
ϵ	total strain
ϵ_c	strain in compressive outer fiber
ϵ_{cr}	creep strain
ϵ_t	strain in tensile outer fiber
μ	Poisson's ratio
ρ	beam curvature
σ	incremental stress
σ_o	stress applied before stress change
τ	time after stress change

Subscripts and Superscripts

i, j, n, r indices

INTRODUCTION

Structural components of vehicles which are subject to aerodynamic heating for sufficient time may undergo significant creep deformation. In recent years considerable research has been carried out in the field of creep on problems involving varying load and temperature conditions (Refs. 1-12), since structural components are generally subjected to unsteady loads and temperatures during service. The approach used in such research has usually been of semi-empirical nature, because relatively large variations in creep conditions fundamentally disturb the internal equilibrium of the materials considered, and render more basic analyses unfeasible. Also many such investigations have been concerned with large values of strain that are generally beyond the range of usual interest for aircraft structures.

In Refs. 13 and 14 results were reported of studies in which materials in tensile creep in the low range of strain at constant temperature underwent incremental changes in stress. The small changes in stress did not disturb the equilibrium of the material appreciably, and a degree of linearity of the material behavior was observed. A more fundamental creep law could thus be ascertained for the effect of the stress increment on creep behavior.

In the present investigation the work in Ref. 14 is being extended by further development of the method of analysis suggested there and application to data obtained from engineering materials. A study is being conducted of primary stage tensile creep of commercially pure aluminum and an aluminum alloy at constant elevated temperature. Material creep data obtained after an incremental change in applied load are used in order to determine the values of

constants appearing in the linear viscoelastic type of relation developed analytically in Ref. 14, and the degree of linearity of the material behavior is thus determined. During creep tests measurements of static modulus are being made and also of the dynamic modulus and internal damping of the material (Ref. 15 and 16), by use of vibration techniques, in order to determine the variation of material properties during the tests and to relate this variation to the shape of the creep curve.

This report summarizes the progress made on the program during the first nine months of work under NASA Research Grant NGR - 52 - 012 - 002. Most of this period was devoted to development of experimental equipment and technique, and preparation of procedures for analysis of data. Details are presented herein, as well as a discussion of some early results obtained from creep tests of commercially pure aluminum specimens.

TEST SPECIMENS

Test specimens were machined from a sheet of commercially pure aluminum of 4mm (0.158 in.) nominal thickness. Results of chemical tests performed on a sample of the sheet show its composition to be as in the following table:

<u>CHEMICAL COMPOSITION OF COMMERCIALY PURE ALUMINUM</u> <u>SHEET MATERIAL</u>						
Element	Fe	Si	Cu	Mn	Zn	Al
Percent by Weight	0.40	0.14	< 0.1	< 0.05	< 0.1	remainder

Dimensions of the test specimens are shown in Figure 1. The reduced section was machined to a nominal width of 12.5mm.(0.493 in.), due care being taken to preserve symmetry and parallelism of the machined edges with respect to the center-line of the specimen. The nominal length of the reduced section was 245 mm. (9.65 in.). The sheet faces of the specimens were not machined. Five holes were provided in the wide end-sections of the specimens for clamping to heavy tension grips. With this type of test specimen the entire reduced section acts as a clamped-clamped beam when lateral force is applied to the beam. (See Ref. 16).

After being machined the specimens were exposed to a temperature of 590°C(1100°F) for approximately two hours, and were then allowed to cool in the annealing furnace. This heat treatment resulted in fully annealed test

specimens. When annealing of the specimens was complete the dimensions of the reduced section of each specimen were carefully measured and a selection was made according to the condition of the faces of the specimens. Specimens whose faces were particularly free of any scratches were reserved for tests in which dynamic measurements were to be included.

TEST EQUIPMENT AND INSTRUMENTATION

The test equipment used in the program consists of a creep-testing machine and a system for applying static and dynamic bending loads to the test specimen. Outputs of the load, strain and temperature transducers are recorded on various autographic recorders.

Creep-Testing Machine

The creep-testing machine shown in Figure 2 was designed to apply deadweight leads of up to 2500 kg.(5500 lb.) in tension with the aid of a beam-loading mechanism. Two points are provided for attachment of the weight-cage, with lever-arm ratios of 20:1 and 5:1. The shorter lever-arm permits more accurate load control at small loads. Hardened knife-edges are used at the pivot of the beam, and at the specimen and weight-cage attachment points. The beam and weight-cage are balanced by a counterweight.

The weight-cage rests on a screw-jack. Load is applied to the test specimen by gradually lowering the jack until the weight-cage hangs free.

The ends of the test specimen are clamped in heavy tension grips,

so that the specimen behaves as a clamped-clamped beam when subjected to lateral vibration. The steel grips, which weight approximately 5.9 kg. (13 lb.) each, as compared to approximately 35 gm. (1.25 oz.) for the specimen, are hinged through their center of gravity, the axis of the hinge lying parallel to the sheet-faces of the specimen. Additional hinged connections are located above the upper grip and below the lower grip, perpendicular to the direction of the grip hinges, in order to permit complete freedom of movement in the horizontal plane. Various parts of the loading linkage were coated with cadmium or aluminum paint for protection against corrosion.

Test Furnace and Control

The tubular furnace used in the creep-testing machine (see Figure 2) is 320 mm. (10 in.) in length, with an internal diameter of 75 mm. (3 in.). The furnace has a capacity of 1000°C (1832°F). Temperature control is achieved by use of a variable bimetallic type of servomechanism, which is inserted through the furnace wall at the midpoint. The controller operates a relay which regulates the power supplied to the three heating elements in the furnace. A rheostat controls the total power to the furnace in accordance with the desired temperature, and power to each of the heating elements can be set proportionately in order to obtain an even temperature distribution along the length of the furnace.

Lateral Loading System

A device used for applying lateral (bending) loads to the test specimen is shown in Figure 3. (The linear variable differential transformer, used for measuring midpoint deflection under lateral loads, has been removed for clarity in the figure). This apparatus, which facilitates determination of static and dynamic moduli and material damping characteristics during creep tests, is mounted on a stand separate from the creep-testing machine so that vibrations cannot be induced in the specimen through the testing machine.

Vibration of the specimen is achieved in the following manner. A brass spring-clip grasps the edges of the specimen at the midpoint, and a thin stainless-steel tubular rod, which is threaded into a small nut welded to the clip, leaves the furnace through a horizontal hole. The clip and rod each weigh approximately 1 gm. A force of 1 kg. (2.2 lb.) is sufficient to detach this assembly from the test specimen. The external end of the forcing rod is attached to a vibrator of 1 kg. capacity by means of a leaf-spring which is lodged in a slit near the external end of the rod. The vibrator is powered by a variable-frequency oscillator of 10 - 100,000 cps capacity, which operates through a 40-watt power amplifier.

The mechanical link formed between vibrator and specimen can be broken for material damping tests by activation of a solenoid which dislodges the leaf-spring from the forcing rod, immediately after the oscilloscope, which measures the vibration decay, is triggered. The triggering circuit is shown in Figure 4.

The loading link with which static bending load is applied to the specimen is essentially similar to that described for vibratory load. A length of copper wire is looped around the test specimen at the lower third-point, and passes horizontally through a hole in the furnace wall. The wire is attached to a flexible cord, which passes over a pulley and from which small weights are hung. The copper wire can be detached from the specimen while damping measurements are being made.

In order to achieve the prescribed clamped end conditions under static bending load, the tension grips are held rigid by auxiliary supports, as shown in Figure 3. Lateral movement of the loading linkage in the direction of the bending load is prevented by bringing two horizontal supports into juxtaposition with tension grips at the hinge line. Rotation of the grips is prevented by inserting locking pins through adjacent linkage into the grips.

Load Cells

Two tensile load cells (Figure 5) were designed and manufactured. The capacity of the smaller load cell (shown in Figure 2 in series with the test specimen) is 125 kg. (330 lb.) and that of the larger load cell is 1250 kg. (3300 lb.). The beam-type load cells are equipped with a foil strain-gage bridge, which gives an output signal of 3.8 mv/v input at capacity load. The amplified output signal is linear within less than 0.5 percent when read on a dc recorder, and hysteresis is negligible. With suitable amplification

of the output signal, load differential of 0.1 percent of capacity load can be measured with ease for both load cells. Before calibration both load cells were subjected to 10,000 cycles of tensile load to 110 percent of capacity load.

Strain Measurement

Both longitudinal and bending strain measurements are obtained by use of two high-temperature wire resistance strain-gages mounted on opposite faces and at the midpoint of the test specimen. A pair of similar gages is mounted on an unloaded strip of aluminum, and placed inside the furnace near the test specimen for temperature compensation of the active arms of the strain-gage bridge (see Figure 6).

Strain gages used in the program are Kyowa type KA-10-A1 high-temperature gages. These gages are of copper-nickel alloy wire and are asbestos-backed. When properly installed and stabilized the gages can be used up to a temperature of 400°C (750°F) for dynamic measurements, and up to a somewhat lower temperature for static and quasi-static tests.

Curing procedure for the silicon cement used for application of the strain gages is as follows:

<u>Step</u>	<u>Temperature</u>		<u>Duration, hr.</u>
	<u>°C</u>	<u>°F</u>	
1	80	175	1
2	120	250	2
3	180	355	2 (followed by gradual cooling)

After the prescribed curing cycle for the cemented gage installation copper-nickel lead wires are spot-welded to the gage leads.

It was found that three or four additional heat cycles of the complete gage and wire system up to the test temperature are required in order to stabilize the strain gages. On the fifth heating of the specimen, and after exposure for approximately one hour, drift of the strain-gage output ceases. Calibration of the strain-gage output against an extensometer (at room temperature) and a dial gage, measuring overall elongation (at room temperature and 200°C), has shown that the gage factor changes less than 2 percent from room temperature to 200°C (392°F), as claimed by the manufacturer.

Strains due to either elongation or bending of the specimen are measured by the single strain-gage bridge. This is achieved by reversing the polarity of half of the bridge, and by the use of two balancing potentiometers, as shown schematically in Figure 7. With the switch in position A the bridge reacts to elongation of the specimen, initial balance being obtained with potentiometer PA. In order to measure strain due to bending at any time during a test, the switch is thrown to position B, with the result that the connections of one active gage and one compensating gage are interchanged and the center

tap of potentiometer PB is connected to the bridge. The bridge in the bending configuration is then balanced by use of potentiometer PB. In this way the zero balance of the elongation configuration (potentiometer PA) is not disturbed and the bridge returns to its original setting when the switch is returned to position A. The effect of switching is less than 3 microinch/inch offset. A calibration resistance can be connected across one of the arms of the elongation bridge by throwing the switch to position C.

Recording of Stress and Strain

The output signal of the load cell is recorded on a multichannel dc oscillograph equipped with high- and medium-gain amplifiers (See Figure 2). The oscillograph is a thermal-pen type and paper travel speeds vary from 0.4 mm (0.016 in.) to 100 mm. (3.93 in.) per sec. The load record can be calibrated directly in units of stress.

The separate systems are used for recording strain, depending on the conditions of the test. In the majority of creep tests the longitudinal strain is recorded on the dc oscillograph. The recorder reaches a sensitivity of 10 microinch/inch/ scale-division with an accuracy of ± 2.5 microinch/inch.

The frequent switching of the automatic temperature control relay of the furnace (every 5 seconds) causes disturbances on the strain record. This difficulty has been circumvented by raising the thermostat setting and reducing the

power supplied to the furnace so that the test temperature is held constant without tripping the thermostat. This procedure requires greater attention by the operator, to ensure by minor adjustments of the power supply that the desired temperature is held. Nevertheless the temperature at a point on the specimen varies less than $\pm 1^{\circ}\text{C}$ at 200°C , and the temperature variation along the test specimen is $\pm 2^{\circ}\text{C}$.

The oscillograph is used for creep tests during which the creep rate is fairly high, the test duration being 10 hours or less. For creep tests conducted for periods of up to one week, at lower creep stress and correspondingly lower creep rate, strain measurements are made on a standard strain-indicator. Other measurements involving small strain readings are also made on the strain-indicator. Such measurements include longitudinal strain due to negative increments in creep load, and strain resulting from static lateral loading.

The dynamic strain signal due to lateral vibration of the test specimen is fed to an oscilloscope through a differential amplifier, and recorded on high speed photographic film (Figure 2). Accurate calibration of the signal is unnecessary, since the record obtained in this case is used for measurements of material damping only. However the signal is calibrated by use of the calibration resistor in the strain gage circuit to ensure that vibratory strains do not exceed 50 microinch/inch peak-to-peak.

Temperature Measurement and Recording

Three pairs of 0.25 mm. diameter (30 gage) iron-constantan thermocouple wire are spot-welded to each specimen. Readings from the thermocouple located

near the midpoint of the specimen are recorded on a strip-chart temperature recorder with a range of up to 700° (1292°F). The remaining two thermocouples are placed approximately 20 mm. (0.8 in.) from either end of the reduced section of the specimen, as shown in Figure 2. Readings from these thermocouples are recorded on the dc oscillograph.

The thermocouple wires as well as the strain-gage lead wires are supported on the aluminum strip on which the compensating gages are installed. In this way negligible mass is added to the specimen by the weight of the wires.

TEST PROCEDURE

Each creep specimen is exposed to the test temperature for one hour prior to test, in order to stabilize the temperature distribution in the furnace and the strain-gage circuit. After exposure, load is applied by lowering the weight-cage with the screw-jack until the cage hangs free. The loading rate is regulated so that a strain rate of approximately .002 in./in./minute is achieved, as recorded on the oscillograph. Simultaneous recording of stress and strain permits accurate determination of the value of the strain at the instant the specimen is fully loaded.

One incremental change in axial load is applied during each creep test at a predetermined time. The load increments never exceed 6 percent of the initial creep load. A positive increment is applied by allowing a prescribed volume of water to drain into a container which has been placed in the weight-cage before the start of the test. The test continues for one hour after the change in load, and is then terminated. Negative load increment is applied by removal of the container, which contains a previously measured quantity of water, from the weight-cage without disturbing the cage. Generally creep strain ceases for some time after the removal of the container of water. The test is allowed to continue until additional creep has occurred for approximately one hour.

When static bending load is applied at various times during a creep test, the procedure is as follows: The strain-gage bridge is connected to a standard

strain-indicator, switched into the bending configuration, and balanced. The grip locking-pins and horizontal supports are brought into position, and a preload of 50 gm. (1.76 oz.) is hung from the loading wire. This causes a strain reading of less than 3 microinch/inch. The purpose of the small preload is to ensure good contact between the grips and the locking pins and supports, without bending the specimen appreciably. A bending load of 500 gm. (1.1 lb.) is then applied to the specimen and the resultant strain and mid-point deflection are recorded. The bending load is immediately removed and the strain and deflection indications are again recorded. It was found that this procedure has no measurable effect on subsequent creep strain or strain rate.

For dynamic measurements the strain-gage output (in the bending configuration) is connected to the calibrated oscilloscope, and the static load wire is detached from the specimen. The specimen is oscillated laterally at the first resonant frequency as determined from the amplitude of the strain signal. Care is taken that the vibratory strain does not exceed 50 microinch/inch peak-to-peak. This magnitude of vibration has no measurable effect on the creep results. The resonant frequency is measured with an accuracy of ± 1 cps by use of an electronic counter connected across the terminals of the variable-frequency oscillator. (See Figure 2)

Material damping measurements are made by photographic recording of the vibration decay appearing on the screen of the oscilloscope when the mechanical link between the vibration and the test specimen is broken. The triggering circuit of the oscilloscope is fired at the same time as the mechanical break is made, as described in the previous section.

LINEAR EQUATION FOR CREEP AFTER INCREMENTAL CHANGE IN STRESS

Perhaps one of the most influential factors which determine the outcome of any analysis of creep is the choice of a stress-strain-time, or creep, equation. In the manner of Ref. 14 a linear operator equation was chosen to describe the effect of an incremental change in stress on creep behavior in the form

$$P(\sigma) = Q(\epsilon) \quad (1)$$

where P and Q are linear operators of the form

$$P = \sum_{r=0}^n p_r \frac{\partial^r}{\partial t^r}, \quad Q = \sum_{r=0}^n q_r \frac{\partial^r}{\partial t^r} \quad \text{respectively,}$$

and σ and ϵ are the increment of stress and the corresponding strain respectively (see Fig. 8). If σ is much smaller than σ_0 , the stress before the stress-change, and Eq. (1) is assumed to hold only in the neighborhood of t_0 , the time at which the stress was changed, then p_r and q_r are functions of σ_0 and t_0 only.

Specifically Eq. (1) determines the additional creep occurring after the stress-change, relative to the creep which would have occurred if the stress had remained at the original level. Eq. (1) is quite general and solutions corresponding to or approximating many empirical creep relations can be obtained from it, depending on the number of terms chosen in the differential equation. For example, if $n = 2$, $q_0 = 0$, and initial conditions are as stated, Eq. (1) may be solved by operational methods (Ref. 17) to give the strain resulting from the stress increment as

$$\epsilon = \sigma \left(\frac{p_1}{q_1} - \frac{p_0 q_2}{q_1^2} \right) + \frac{p_0}{q_1} (t - t_0) + \left(\frac{p_2}{q_2} + \frac{p_0 q_2}{q_1^2} - \frac{p_1}{q_1} \right) \exp \left[- \frac{q_1}{q_2} (t - t_0) \right] \quad (2)$$

as obtained in Ref. 14. Eq.(2) is the simplest solution obtainable for the initial conditions considered, since solutions of Eq. (1) with $n = 3$ or $q_0 = 0$ (or both) contain both trigonometric and hyperbolic functions, which are difficult to apply to the normal creep curve.

It will be noted in Eq. (2) that strain is directly linear with the stress; that is, doubling of the stress will result in doubling the strain. Although this characteristic is convenient from the mathematical standpoint, linear behavior of this type in engineering materials might be expected to be the exception rather than the rule. For cases in which the linear solution proves inadequate, agreement between theory and experiment may be improved by substituting a suitable function of stress, $f(\sigma)$, for σ in Eq. (1), so that

$$P [f(\sigma)] = Q(\epsilon) \quad (3)$$

In the case of a constant increment of stress, for example, the function $f(\sigma)$ is independent of time with the result that the Laplace transforms of $f(\sigma)$ and its derivatives with respect to time contain the function $f(\sigma)$ in the first degree explicitly. Thus $f(\sigma)$ may be factored out of the transformed equation and the solution Eq. (2) is again obtained, except that $f(\sigma)$ must be substituted for σ . This procedure introduces nonlinearity of the creep behavior along the stress axis into the analysis.

METHODS OF ANALYSIS OF EXPERIMENTAL DATA

As an initial approach to the analysis of experimental creep data, the material creep after an incremental change in stress was assumed to be a linear function of the stress increment. Therefore Eq. (2) could be taken as the creep relation, so that data reduction involves determination of the values of the constants in the equation, followed by investigation of the degree of linearity observed in the experimental results. The procedure developed for determining the constants will be described in this section. Methods used for determination of static and dynamic moduli as well as material damping characteristics, from data obtained in static and dynamic bending tests during creep, will also be presented. Extensive use is made of the electronic computer for most of the calculations.

Determination of Creep Constants

Values of the creep constants appearing in Eq. (2) are calculated from the amount of strain due to the incremental stress-change σ at time t_0 . This amount of strain is determined from the test data as the total strain minus the strain at the basic stress σ_0 , extrapolated beyond t_0 . The method of extrapolation of creep data is critical when the creep strain involved is on the order of 0.5 percent or less, as in the present instance. The extrapolation technique is especially important when the change in stress is small, causing a correspondingly small effect on the strain. Therefore it is necessary to fit the creep data obtained under σ_0 to a suitable empirical expression for purposes of extrapolation.

Reference to the typical creep curve shown to logarithmic scale in Figure 9 indicates that a suitable empirical relation describing the creep strain ϵ_{cr} before change of stress is of the form

$$\epsilon_{cr} = at^b \quad (4)$$

where a and b are constants. The empirical constants are determined for each creep test by the method of least squares. It is of interest to note that the average value of b obtained from 12 creep tests at 200°C (392°F) was 0.35, the scatter band being ± 0.046 . The value obtained for b is in good agreement with the classical value of $1/3$ (Ref. 18). Extrapolation of the creep curve in accordance with Eq. (4) beyond time t_o permits the determination of the additional strain caused by the stress increment applied at t_o , and thence the value of constants in Eq. (2).

The form of Eq. (2) is not amenable to direct machine computation of the constants. However these may be conveniently computed by use of a process of graphical differentiation. The procedure, which is described in Ref. 19, is as follows:

Eq. (2) may be written in the form

$$\epsilon = k_o + k_1\tau - k_2 \exp(-k_3\tau) \quad (5)$$

where

$$k_o = \frac{p_1}{q_1} - \frac{p_o q_2}{q_1^2}$$
$$k_1 = \frac{p_o}{q_1}$$

$$k_2 = \frac{p_1}{q_1} - \frac{p_2}{q^2} - \frac{p_0 p_2}{q_1^2}$$

$$k_3 = \frac{q_1}{q_2}$$

and

$$\tau = t - t_0$$

Differentiation of Eq. (5) twice with respect to τ yields the expression

$$\frac{d^2 \epsilon}{d\tau^2} = -k_2 k_3^2 (\epsilon - k_0 - k_1 \tau) \quad (6)$$

or by substitution in Eq. (5):

$$\frac{d^2 \epsilon}{d\tau^2} = k_3^2 (\epsilon - k_0 - k_1 \tau) \quad (7)$$

The constants k_0 , k_1 and k_3 may be calculated from Eq. (7) by the method of least squares. The remaining constant k_2 is then determined by substituting the values of k_0 , k_1 and k_3 into Eq. (5) and by again applying the method of least squares. However, in order to solve for the constants in Eq. (7), $\frac{d^2 \epsilon}{d\tau^2}$ must be determined against τ from the experimental data.

If the strains ϵ_i , corresponding to times τ_i , are given at constant time intervals, that is

$$\tau_{i+1} - \tau_i = \Delta\tau \quad (8)$$

then the second derivative of any ϵ_i with respect to time may be obtained by differentiation of the Stirling interpolation formula (Ref. 19):

$$\frac{d^2 \epsilon_i}{d\tau^2} = \frac{1}{(\Delta\tau)^2} \left[(F_{1i} - 2\epsilon_i) - \frac{1}{12}(F_{2i} - 4F_{1i} + 6\epsilon_i) + \frac{1}{90}(F_{3i} - 6F_{2i} + 15F_{1i} - 20\epsilon_i) + \dots \right] \quad (9)$$

in which the terms F_{ji} are defined as

$$F_{ji} = \epsilon_{i-j} + \epsilon_{i+j} \quad (10)$$

For conservative functions Eq.(9) converges rapidly for a suitable choice of $\Delta\tau$ and the first three terms of the series, as presented here, are generally sufficient for purposes of computation. Note, however, that in Eq. (9) the third term on the right is not defined at the first three and last three data points. Depending on the data being analyzed this deficiency could be a cause for concern, although no difficulties have arisen in the processing of creep data in the present investigation. Possible errors due to loss of end points in the graphical differentiation are corrected when k_2 is subsequently evaluated from Eq. (5).

Determination of Dynamic Modulus During Creep

The dynamic modulus of a material may be determined by measuring the resonant frequency of vibration of a beam made of the material. For a beam with clamped ends and rectangular cross-section, the resonant frequency is given by the relation

$$f_n = 0.288 C \frac{h}{l^2} \sqrt{\frac{E_d g}{w}} \quad (11)$$

where f_n is the resonant frequency (in cps) for mode $n = 1, 2, 3, \dots$

h is the beam thickness in the direction of vibration

l is the beam length

E_d is the dynamic modulus

g is the acceleration of gravity

w is the specific weight of the beam material

and C is a constant dependent on the vibrational mode n and on the end conditions of the beam. For a clamped-clamped beam and $n = 1$, for example, $C = 3.58$; for $n = 3$, $C = 19.2$. Thus the dynamic modulus is given by the equation

$$E_d = \left(\frac{0.288C f_n^2 l^2}{h} \right)^2 \frac{w}{g} \quad (12)$$

During creep at elevated-temperature the length and width of the beam change with temperature and strain. The length is defined as

$$l = l_0 (1 + \alpha \Delta T) (1 + \epsilon) \quad (13)$$

and the width as

$$h = h_0 (1 + \alpha \Delta T) (1 - \mu \epsilon) \quad (14)$$

in which

l_0 and h_0 are the initial length and width, respectively, at room temperature

α is the coefficient of linear expansion

ΔT is the difference between the test temperature and room temperature

ϵ is the total longitudinal strain

and

μ is Poisson's ratio.

When the change in length and width is taken into account, and terms of second and higher order are neglected, Eq. (12) becomes

$$E_d = \left(\frac{0.288 C f_n l_o^2}{h_o} \right)^2 \frac{w}{g} \left(\frac{1 + 2\alpha\Delta T + 4\epsilon}{1 - 2\mu\epsilon} \right) \quad (15)$$

Generally the Poisson effect can also be neglected.

Measurement of Material Damping

When the resonance peak is fairly sharp, as is the case in the work presently in progress, material damping is measured most conveniently by photographing a record of the vibration decay occurring at resonant frequency of the specimen when the driving force is removed. The damping coefficient δ is then given as the logarithmic decrement by the equation

$$\delta = \frac{\ln \frac{A_o}{A_n}}{n} \quad (16)$$

where A_o and A_n are the amplitudes of vibration n cycles apart. (Ref 20).

Determination of Static Modulus During Creep

Evidence exists (Ref. 14) indicating that the static modulus of a material for increasing stress may decrease significantly during creep. If such is the case in general, determination of the variation would have an

important bearing on the results of creep-buckling theories. The static modulus can be obtained by momentarily applying a small bending moment to the creep specimen and measuring the strain and deflection due to bending. If the static modulus does indeed decrease during creep, deformation due to the superimposed bending moment will correspond to a combination of two material stiffnesses. The side of the specimen which is subjected to negative bending stress will unload and deform along the elastic modulus line. The side subjected to positive bending stress will deform in accordance with the static modulus at the creep stress. With the additional assumption that plane sections remain plane after bending, a reduced or effective modulus E_R for a beam of rectangular cross-section is obtained in the form

$$E_R = \frac{4EE_{\text{creep}}}{(\sqrt{E} + \sqrt{E_{\text{creep}}})^2} \quad (17)$$

where E is Young's modulus and E_{creep} is the modulus for increasing creep stress. Eq. (17) corresponds to the reduced modulus obtained by von Karman for the inelastic beam (Ref. 21). The reduced modulus E_R is related to the bending moment M by the equation

$$M = - E_R I / \rho \quad (18a)$$

$$= \frac{E_R I}{h} (\epsilon_t - \epsilon_c) \quad (18b)$$

In Eq. (18) I is the moment of inertia of the specimen cross-section, ρ is the curvature, and ϵ_t and ϵ_c are the outer-fiber strains corresponding to the tensile bending and compressive bending stresses respectively. Thus the modulus during creep, E_{creep} , can be obtained by use of Eqs. (17) and (18b) in conjunction with measurements of the instantaneous strain $(\epsilon_t - \epsilon_c)$ induced by the bending moment M , which can be calculated.

The bending moment applied to the test specimen may be ascertained by conventional virtual work and superposition methods (Ref. 22). With reference to Figure 10, if $x_1 = l/3$, the bending moment at the midpoint of the beam is given by the relation

$$M_{x=l/2} = \frac{Wl}{18} - Hy_{x=l/2} \quad (19)$$

where W and F are lateral and tensile loads respectively, and $y_{x=l/2}$ is the deflection of the beam specimen, experimentally measured at the midpoint.

PRELIMINARY EXPERIMENTAL RESULTS

In this section results of a number of early creep tests conducted in the program will be presented. Since these results represent a small part of the data being generated in the program, the discussion of the results will be minimal and no general conclusions should be drawn at this juncture.

Stress-Strain Behavior

Typical stress-strain curves obtained from commercially pure aluminum specimens, in the fully annealed condition, are shown in Figure 11. Values of elastic modulus were $7,000 \text{ Kg/mm}^2$ ($10 \times 10^6 \text{ psi}$) at room temperature and $6,500 \text{ Kg/mm}^2$ ($9.25 \times 10^6 \text{ psi}$) at 200°C (392°F). Yield stress was found to be 2.16 Kg/mm^2 ($3,060 \text{ psi}$) at room temperature and 1.75 Kg/mm^2 ($2,480 \text{ psi}$) at 200°C . At room temperature the ultimate strength was 7.45 Kg/mm^2 ($10,600 \text{ psi}$) with a total elongation of 55 percent.

Creep under Basic Stress

As mentioned earlier the creep curve under the basic stress, i.e. before change of stress, at a temperature of 200°C is well characterized by Eq. (4) (see Figure 9). Values of the constant b were approximately 0.35, and the value of constant a appeared to be an exponential function of the stress, as often found in the literature (e.g. Ref. 11).

Creep after Incremental Change in Stress

Preliminary results of creep data, obtained from the specimens reported on in the previous paragraph, after small positive and negative changes in tensile stress are presented in Figures 12 and 13 respectively. Terminology used in the figures corresponds to that of Figure 8. Results are plotted in terms of the creep compliance, that is, net strain caused by the stress increment, divided by the absolute value of the stress increment, against time after the change in stress. In all test results shown in Figures 12 and 13 the time at which the stress was changed was 5 hours.

The creep compliances obtained after positive change in stress (Figure 12) appear to fall on a single curve, within the limits of experimental scatter. Results indicate that, at least for stress increments up to 6 percent of the basic stress, the strain is linear with stress increment for this material. The dependence of the constants in Eq. (2) on basic stress, time of stress change and the test temperature remains to be determined.

Creep compliances obtained after negative change in stress (Figure 13) do not appear to substantiate Eq. (2) in their present form. The behavior after the stress change is characterized by a period of recovery during which negligible creep occurs. At a later time, dependent on the magnitude of the change in stress, the creep process is resumed. Results shown in Figure (13) have been replotted in Figure (14) with the omission of the recovery time. The curves of

Figure (14) now appear to show linear dependence on the stress decrement, in agreement with Eq. (2), although the experimental scatter is somewhat broader than for positive stress changes.

The dependence of recovery time after negative stress change on the stress decrement is presented in Figure 15. Preliminary results indicate that at constant temperature the recovery time varies inversely with the basic stress and is a logarithmic function of the stress decrement. An analytical description of the total effect of negative stress change, including the recovery time, is part of future work in the research program. The analysis is based on a model of dislocation motion in the material, as proposed in the literature (Ref. 23).

CONCLUDING REMARKS

Experimental equipment and procedures have been described, and methods used in analysis of the data obtained have been presented. Preliminary creep results obtained from commercially pure aluminum sheet at 200°C indicate behavior after an incremental stress change in accordance with a linear viscoelastic type of analysis.

Work presently in progress is directed towards determination of the dependence of the constants appearing in the analytical equations on the value of the basic stress, the time of stress change and the test temperature. A similar investigation as that being carried out with commercially pure aluminum will be conducted on AU-4GI aluminum-alloy sheet material.

REFERENCES

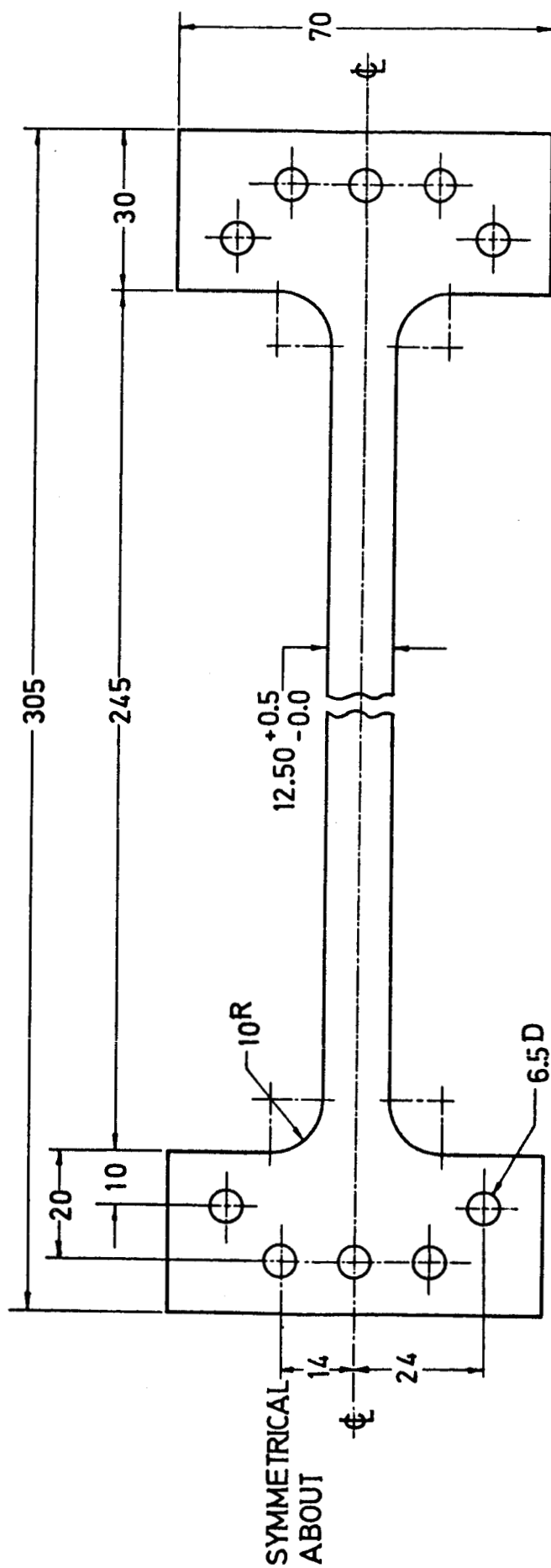
1. Manson, S.S., & Brown, W.F., Jr., Proc. Fourth Sagamore Ord. Materials Research Conf. - "High Temperature Materials, Their Strength Potentials & Limitations", 282-338, Syracuse Univ. Res. Inst., 1957.
2. Isäksso, A., "Krypning vid Variabel Spänning och Temperatur Bibliographi. (Bibliography on Creep under Variable Stress & Temperature)." Roy. Inst. of Tech., Sweden, Publication No. 116, 1957..
3. Proc. of Joint International Conference on Creep 1, Inst. of Mechanical Eng., London, 1963.
4. Finnie, I., & Heller, W.R., " Creep of Engineering Materials" Ch. 10, McGraw-Hill Book Co., Inc., New York, 1959.
5. Odqvist, F.K.G., & Hult, J., " Kriechfestigkeit metallischer Werkstoffe, Ch. III, Springer, Berlin, 1962.
6. Kennedy, A.J., " Processes of Creep and Fatigue in Metals", John Wiley & Sons, New York, 1963.
7. Taira, S., Proc. of the IUTAM Colloquium on Creep in Structures, p. 96, Springer, Berlin, 1962.
8. Garofalo, F., " Fundamentals of Creep and Creep-Rupture in Metals" Macmillan, New York, 1963.
9. Namestnikov, V.S., J. Appl. Mech. and Tech. Phys. (U.S.S.R.), 2 (1964), 180.

10. Jakewluk, A. & Ziemba, S., Experimental Mechanics, 6 (1966), 511.
11. Berkovits, A., " Investigation of Three Analytical Hypotheses for Determining Material Creep Behavior under Varied Loads, with an Application to 2024-T3 Aluminum-Alloy Sheet in Tension at 400°F, NASA TN D-799, 1961.
12. Berkovits, A., " Determination of the Creep Deflection and Lifetime of Aluminum-Alloy Multiweb Box Beams Subjected to Varied Loads at Constant Temperature", NASA TN D-1265, 1962.
13. Wyatt, O.H., Proc. Phys. Soc. London, B 66, (1953), 495.
14. Onat, E.T., & Wang, T.T., Proc. of the IUTAM Colloquium on Creep in Structures, p. 125, Springer, Berlin, 1962.
15. Bodner, S.R., & Amba Rao, C.L., "Experimental Study of the Dynamic Modulus of Aluminum in the Presence of Creep", (Nonr-562 (20)/32) Div. of Engineering, Brown University, Providence, R.I., 1962.
16. Clark, J.W., & Hagel, W.C., Trans. of the ASM 52 (1960), 95.
17. Churchill, R.V., "Modern Operational Mathematics in Engineering", McGraw-Hill Book Co., Inc., New York, 1944.
18. Andrade, E.N. da C., Proc. Roy. Soc. A., 84 (1910), 1.
19. Kopal, Z., "Numerical Analysis" 2nd Ed., Chapman and Hall, London, 1961.
20. Metallurgia 69, (1964), 247-249.

21. Timoshenko, S.P. and Gere, J.M., " Theory of Elastic Stability",
2nd Ed., McGraw-Hill, New York, 1961.
22. Hoff, N.J., "Analysis of Structures", John Wiley & Son, New-York, 1956.
23. Johnston, W.G., J. Appl. Phys., 33 (1962), 2716.

ACKNOWLEDGEMENT

The author is indebted to Professor S.R. Bodner of the Material Mechanics Laboratory, Technion, for invaluable discussions and advice, and to A. Grunwald and A. Davidovits for their assistance in the laboratory.



ALL DIMENSIONS MILLIMETERS
TOLERANCE (UNLESS INDICATED) : 0.25 mm

FIG. 1 CREEP TEST SPECIMEN

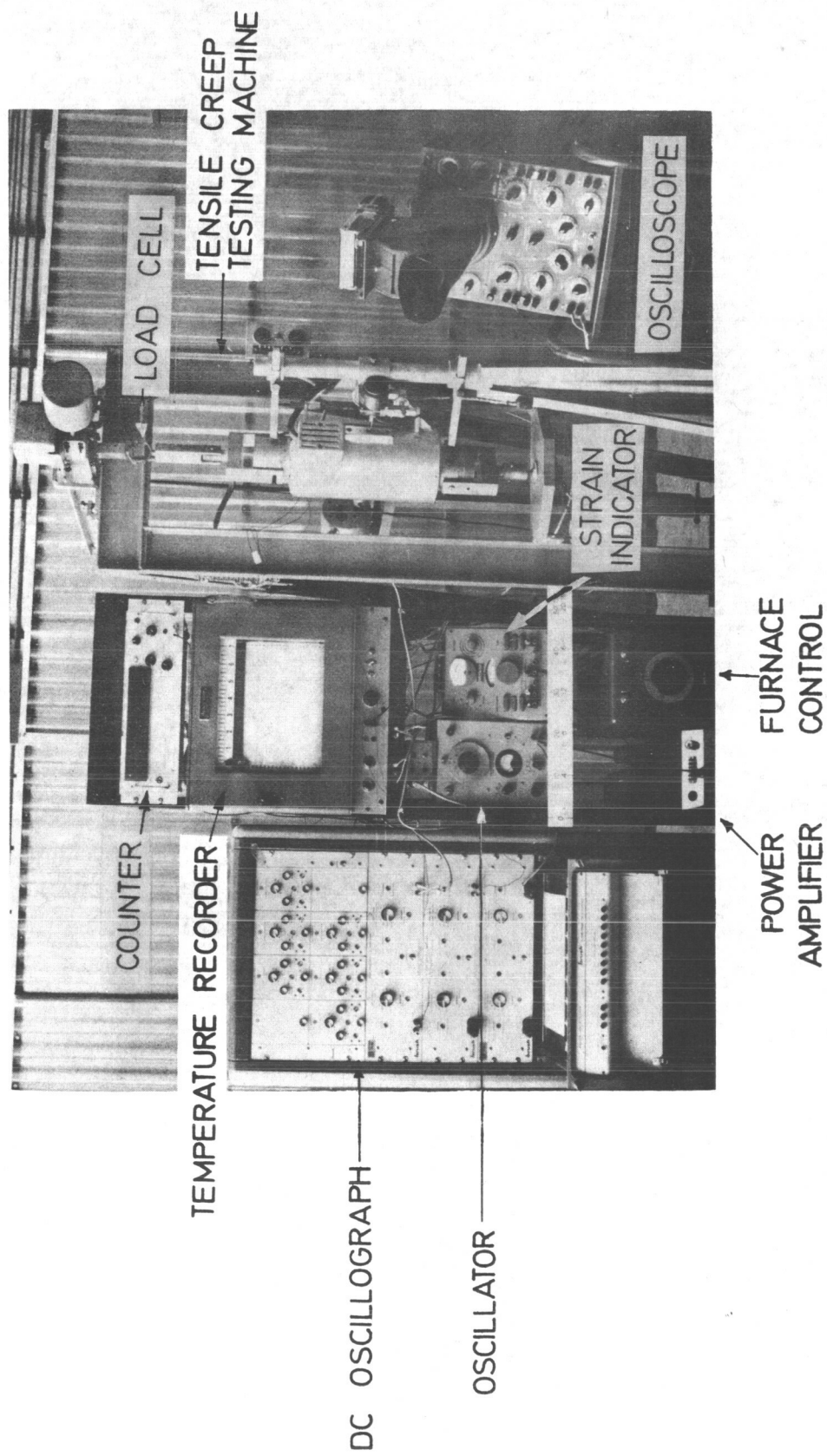


FIG. 2 GENERAL VIEW OF EXPERIMENTAL SET - UP

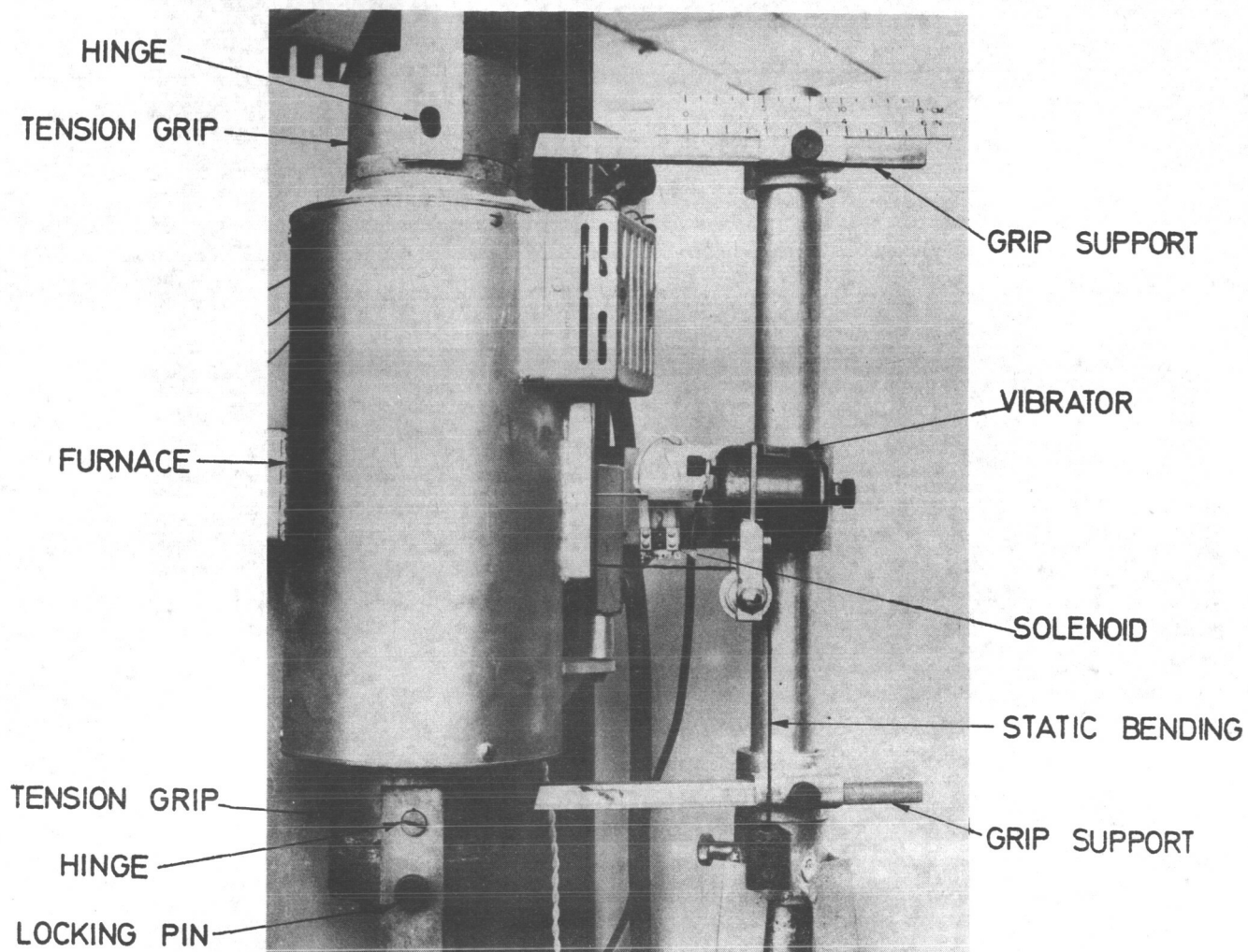


FIG. 3 LATERAL LOADING ASSEMBLY

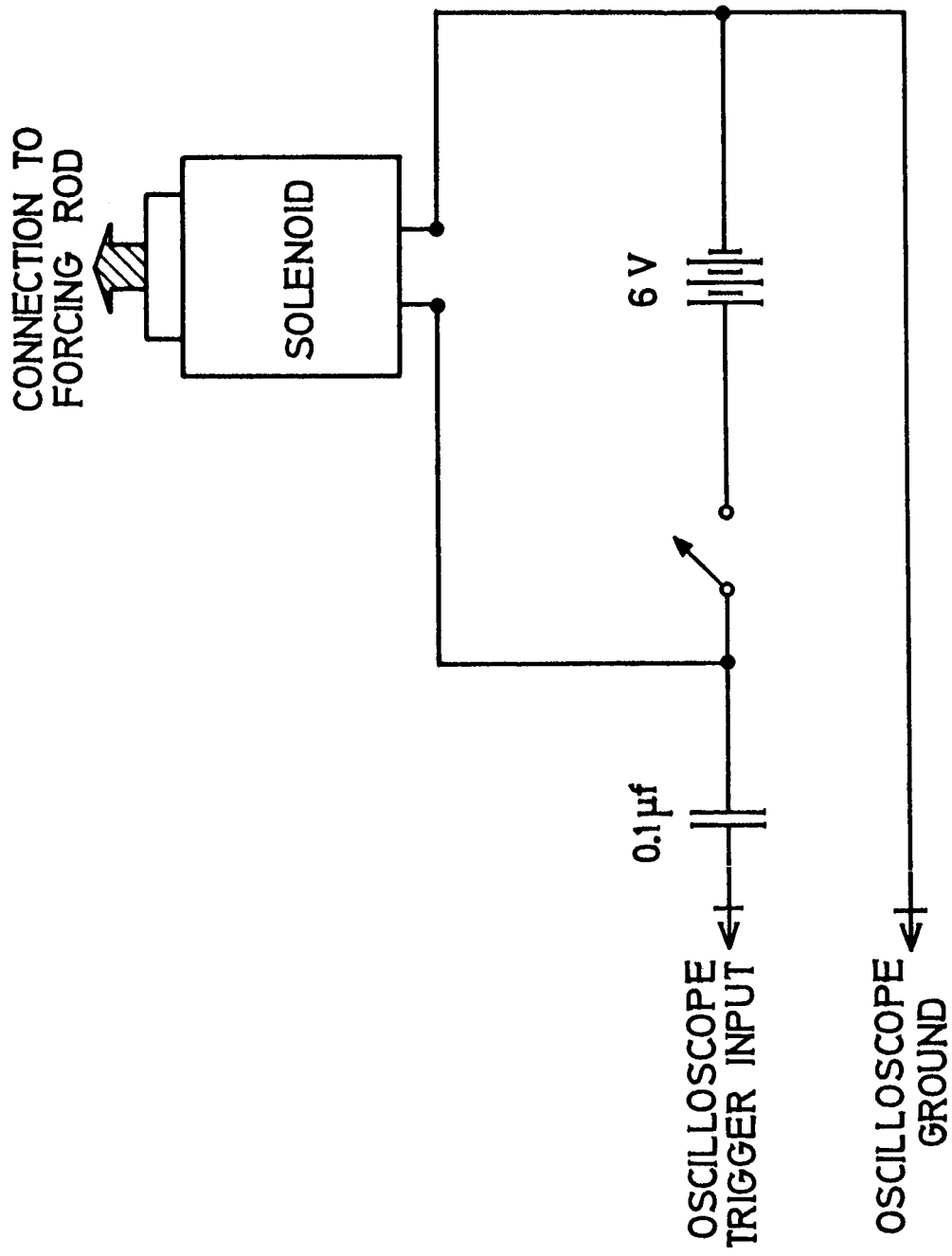


FIG. 4 CIRCUIT FOR TRIGGERING OSCILLOSCOPE AND DISCONNECTING VIBRATOR FROM SPECIMEN

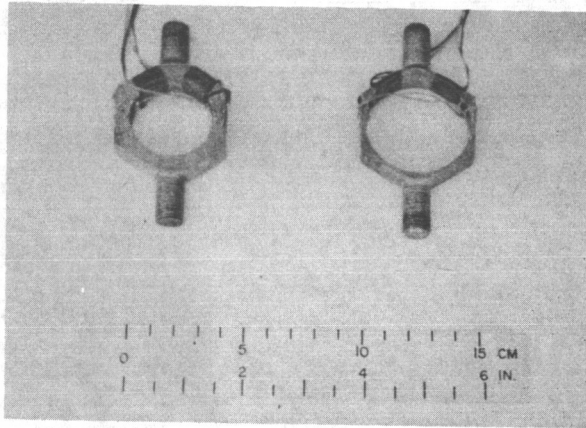


FIG. 5 1250 KG AND 125 KG CAPACITY LOAD CELLS

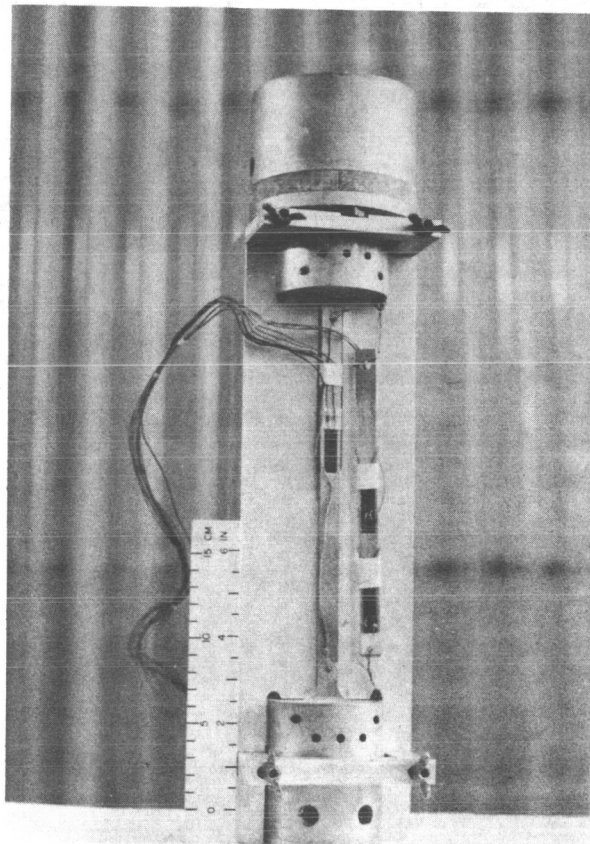


FIG. 6 INSTRUMENTED TEST SPECIMEN IN TENSION GRIPS

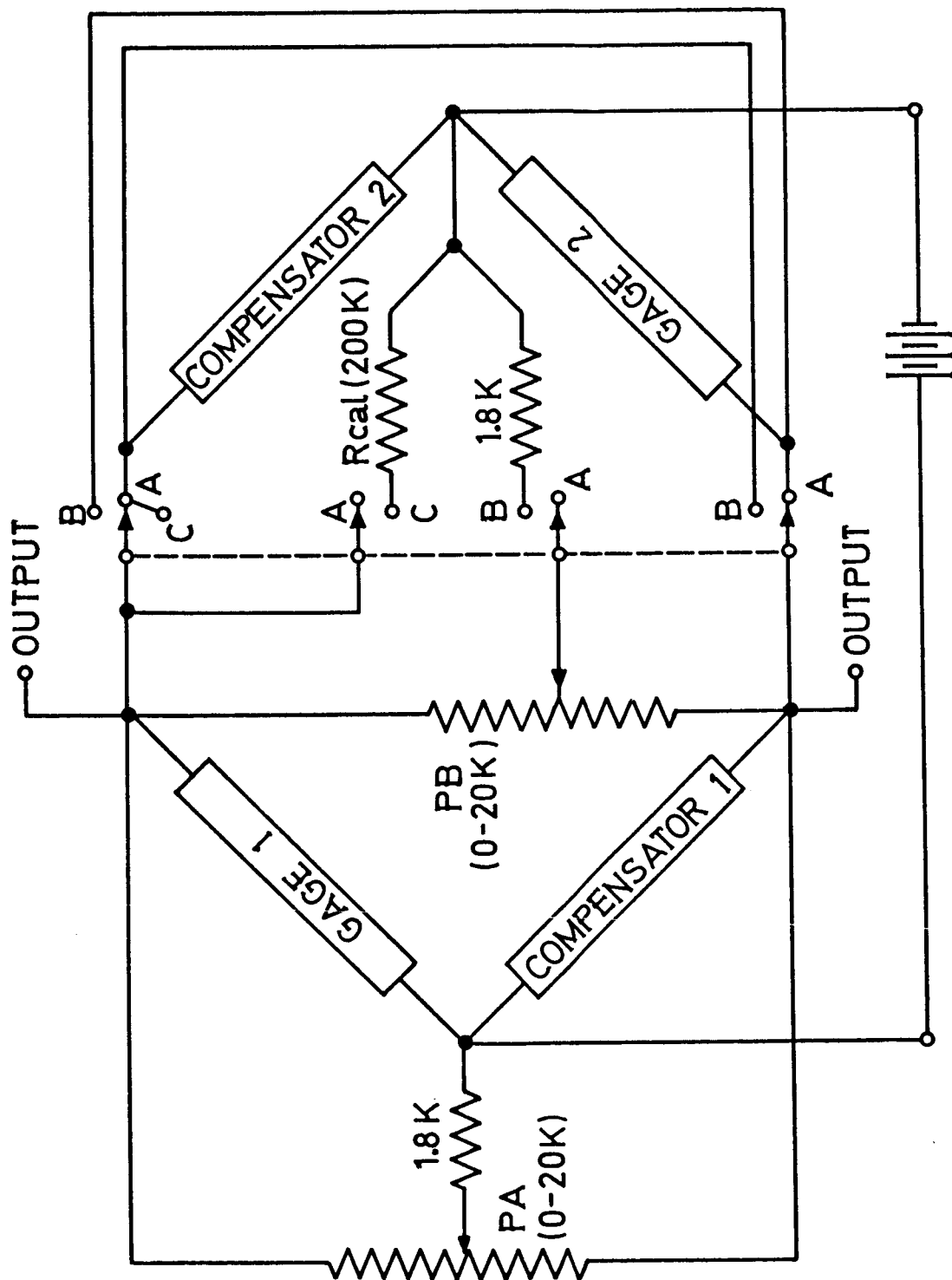


FIG. 7 STRAIN-GAGE BRIDGE CIRCUIT

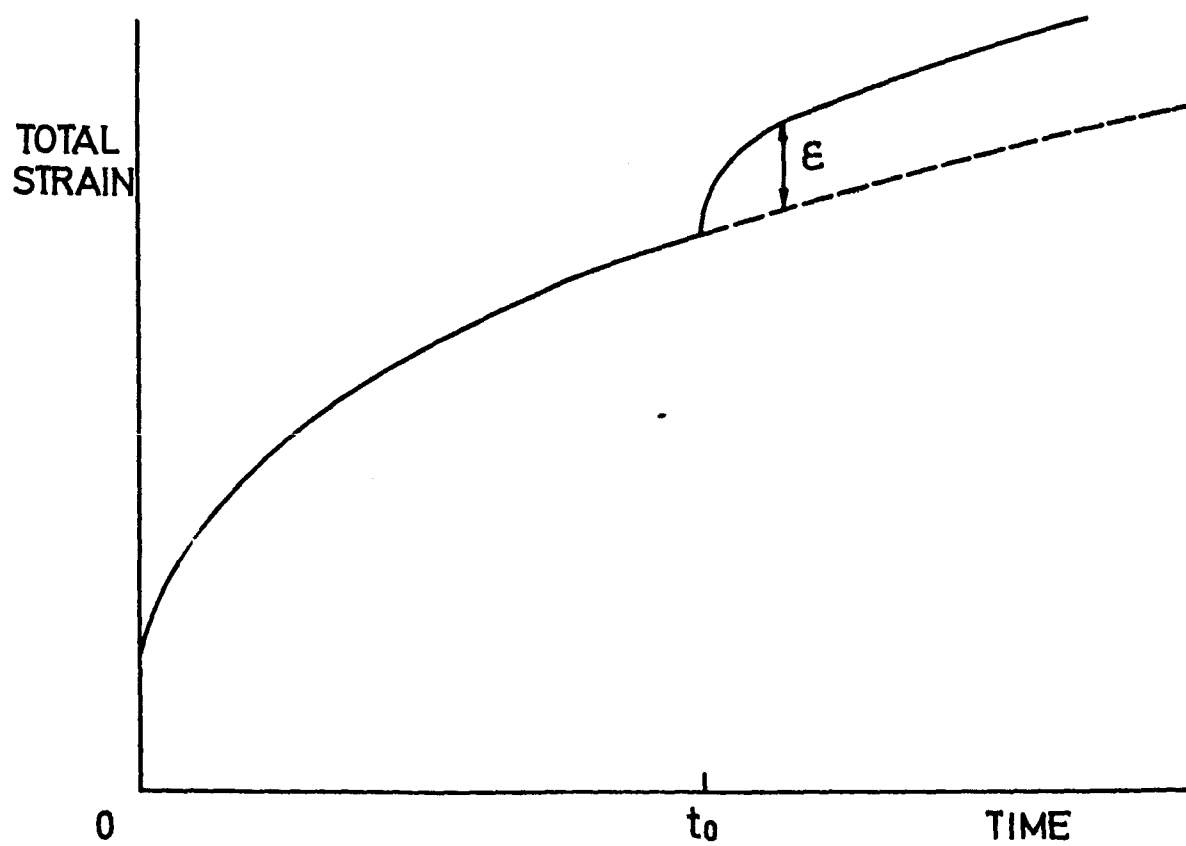
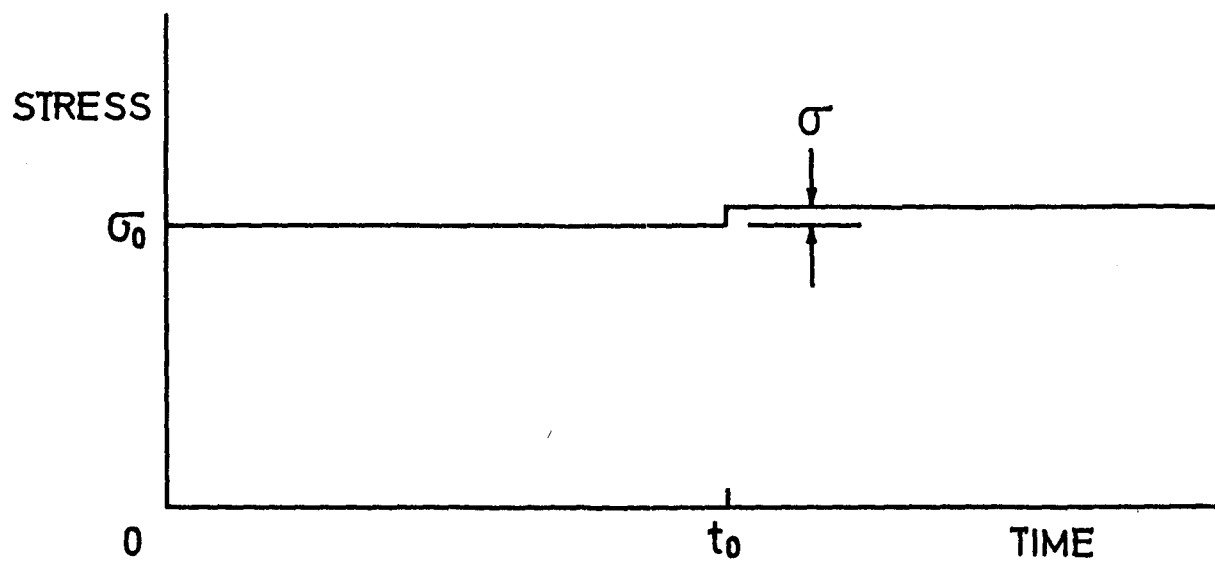


FIG. 8 EFFECT OF INCREMENTAL CHANGE IN STRESS ON CREEP BEHAVIOR

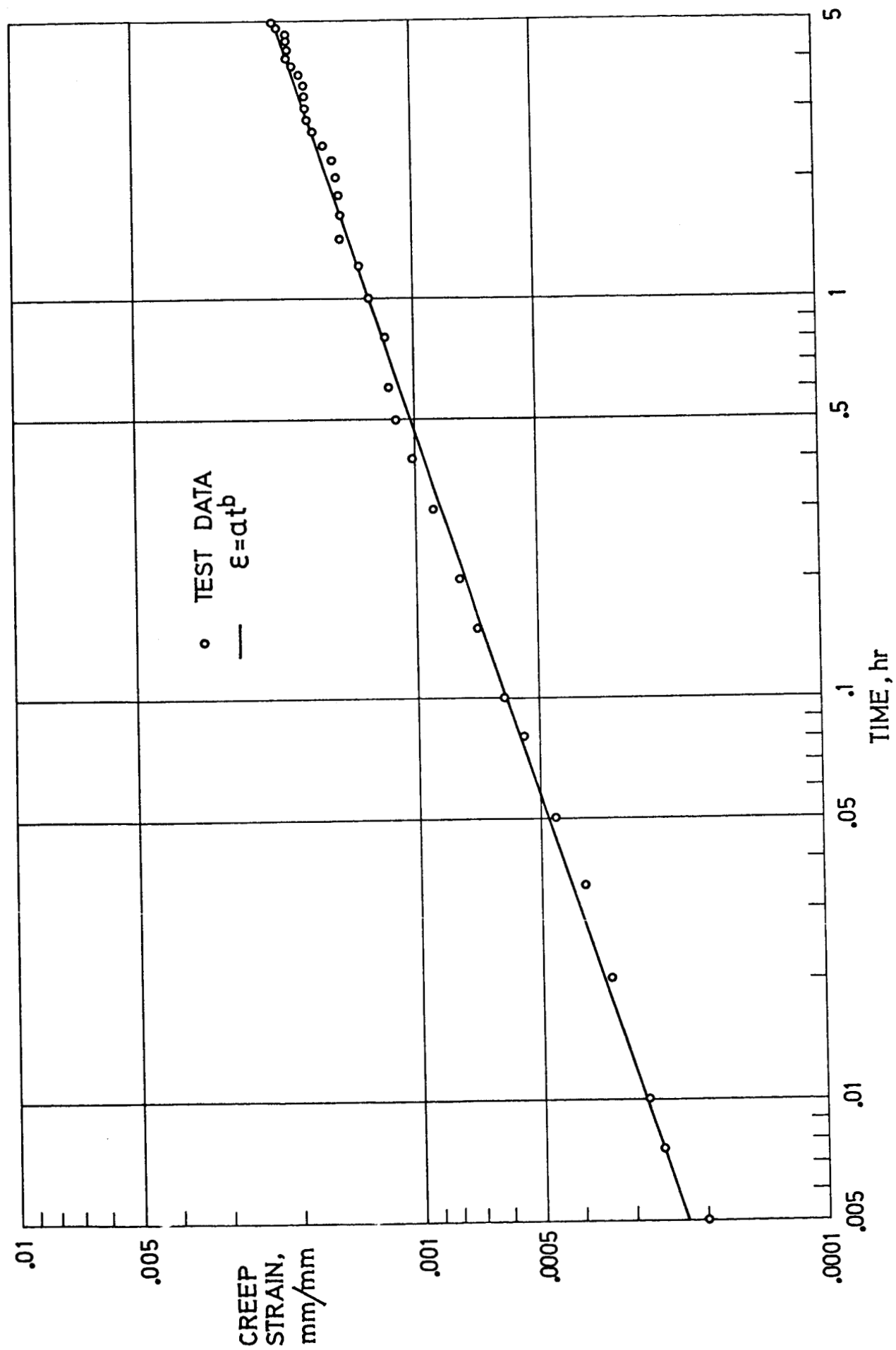


FIG. 9 CREEP CURVE FOR SPECIMEN A57 RESULTING FROM CURVE-FITTING PROCEDURE

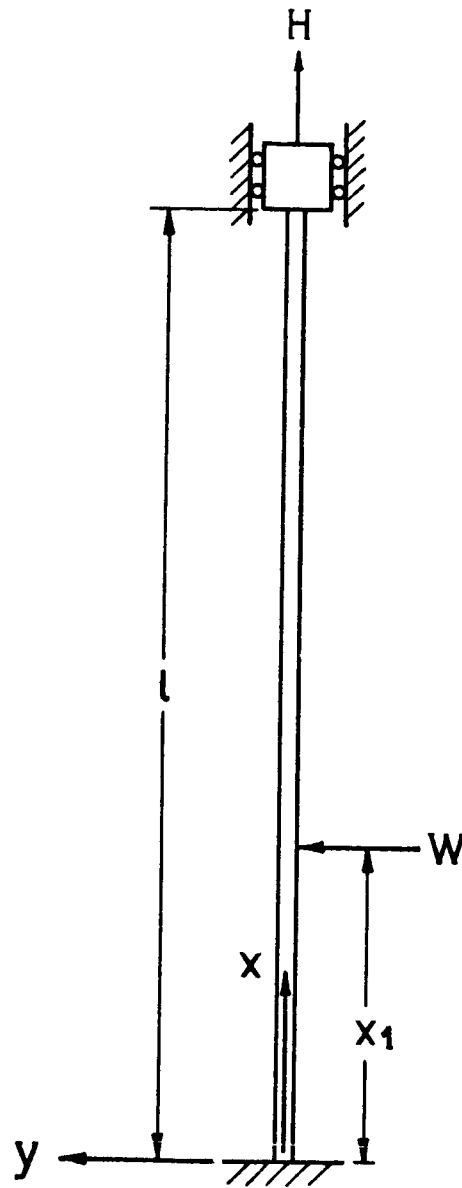


FIG. 10 BEAM ON FIXED SUPPORTS AND SUBJECTED TO CONCENTRATED BENDING AND TENSILE FORCES

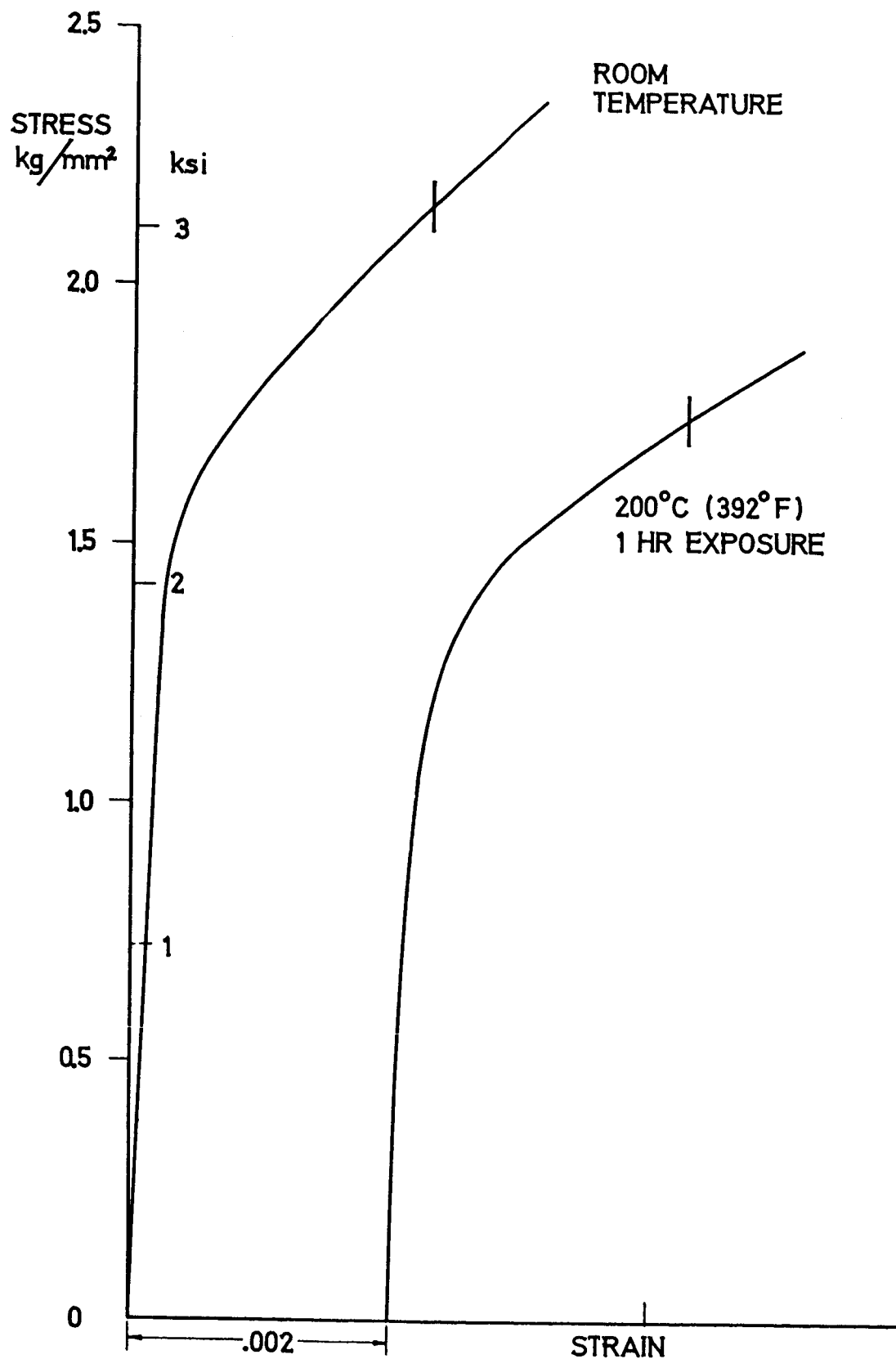
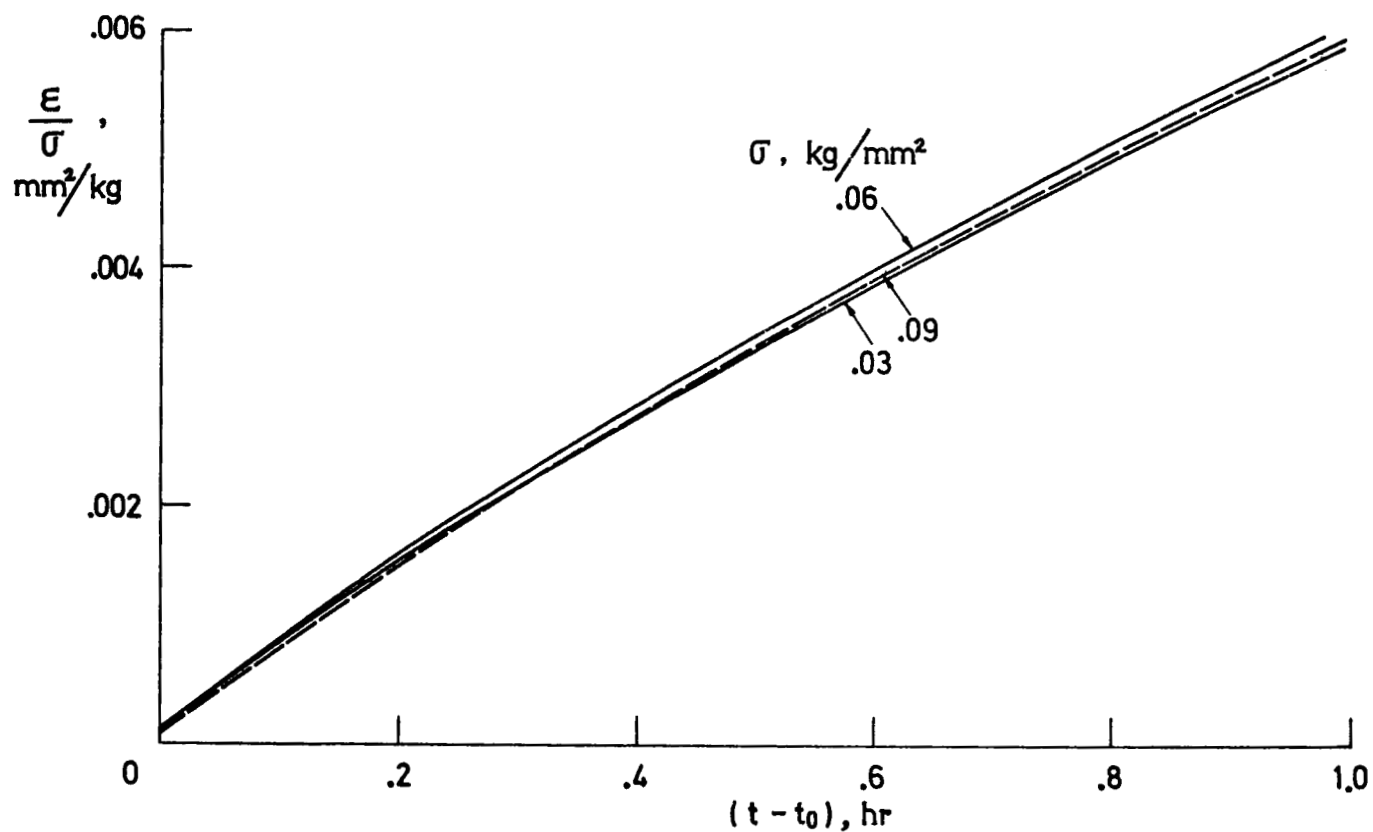
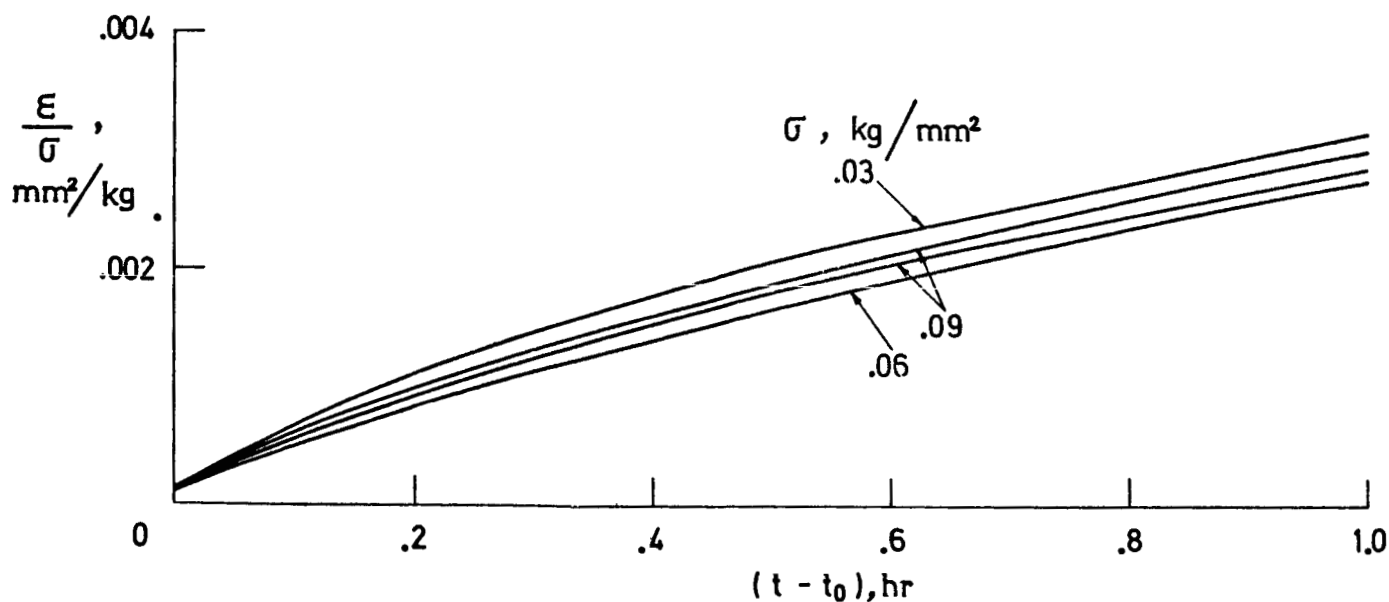


FIG. 11 TENSILE STRESS-STRAIN CURVES FOR ANNEALED
COMMERCIALLY PURE ALUMINUM

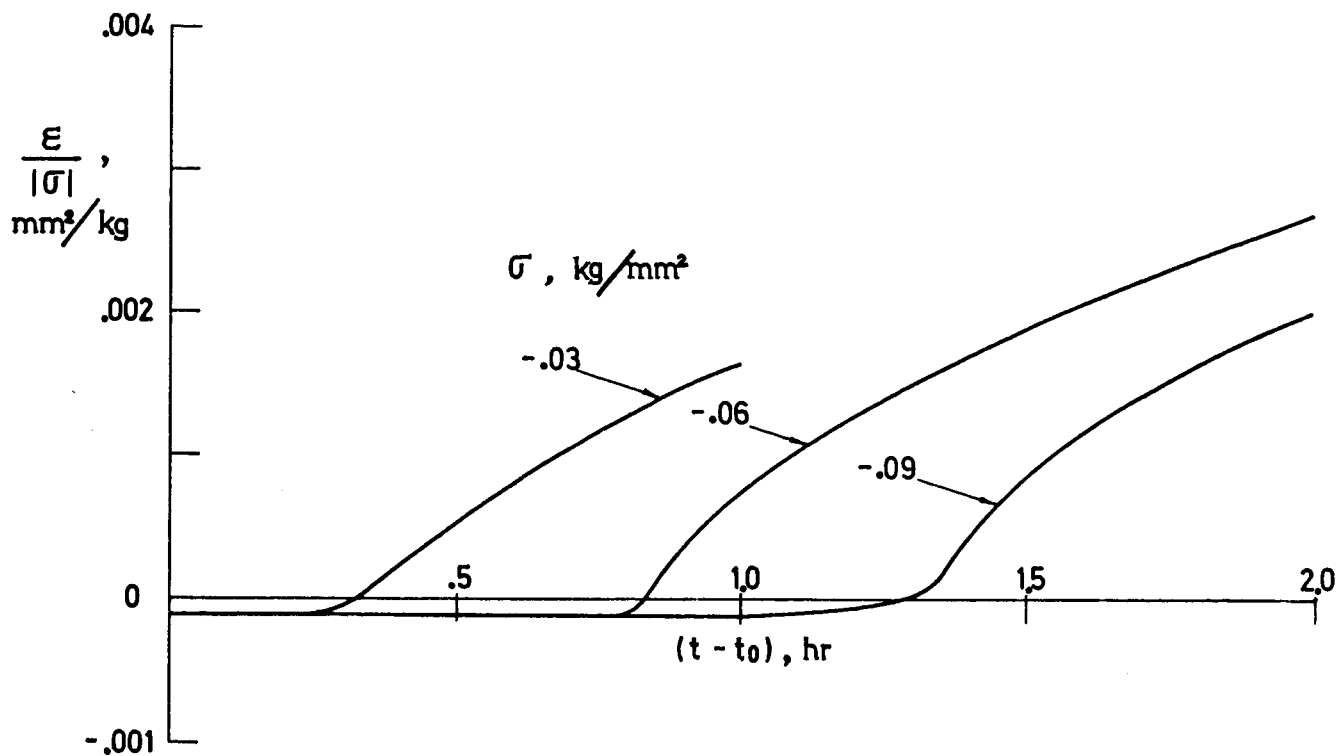


a) $\sigma_0 = 1.62 \text{ kg}/\text{mm}^2$ $t_0 = 5 \text{ hr}$

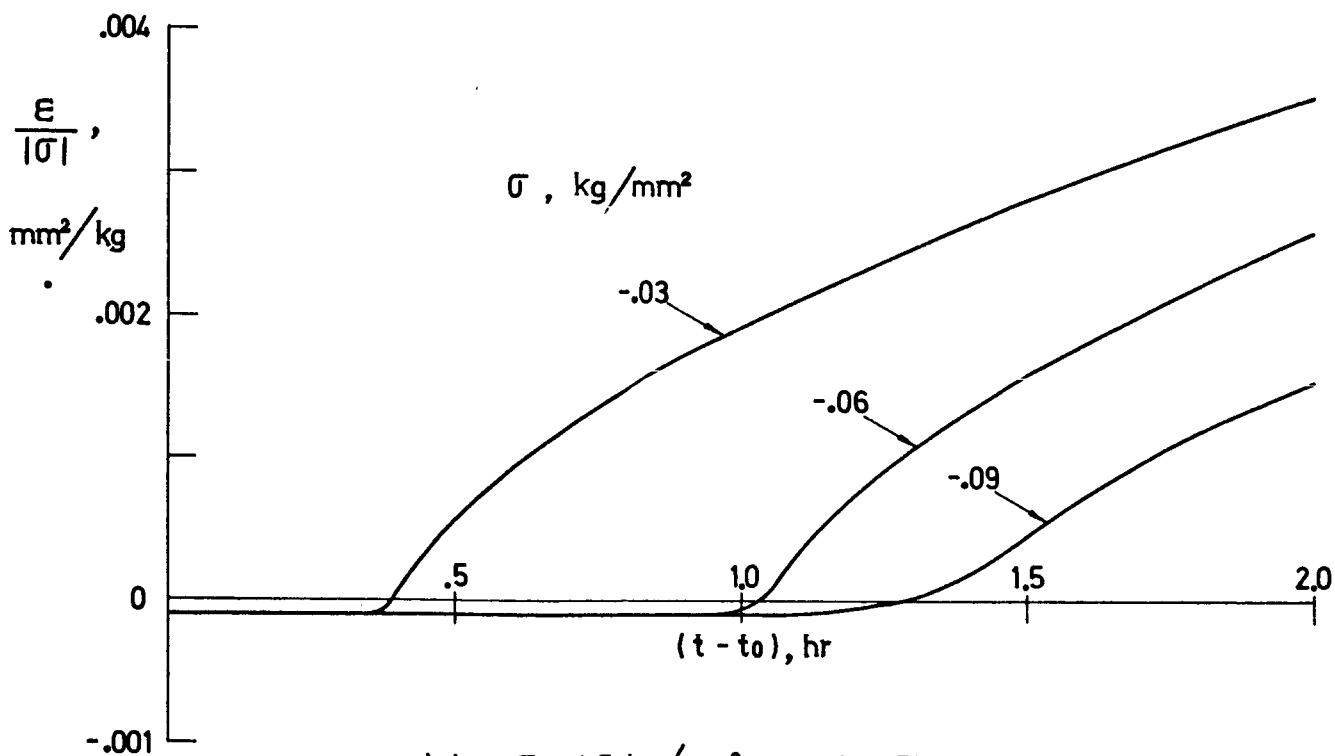


b) $\sigma_0 = 1.5 \text{ kg}/\text{mm}$ $t_0 = 5 \text{ hr}$

FIG. 12 CREEP OF COMMERCIAL PURE ALUMINUM SHEET AT 200°C AFTER INCREMENTAL CHANGE IN TENSILE STRESS



a) $\sigma_0 = 1.62 \text{ kg/mm}^2$ $t_0 = 5 \text{ hr}$



b) $\sigma_0 = 1.5 \text{ kg/mm}^2$ $t_0 = 5 \text{ hr}$

FIG. 13 CREEP OF COMMERCIAL PURE ALUMINUM SHEET AT 200°C AFTER DECREMENTAL CHANGE IN TENSILE STRESS

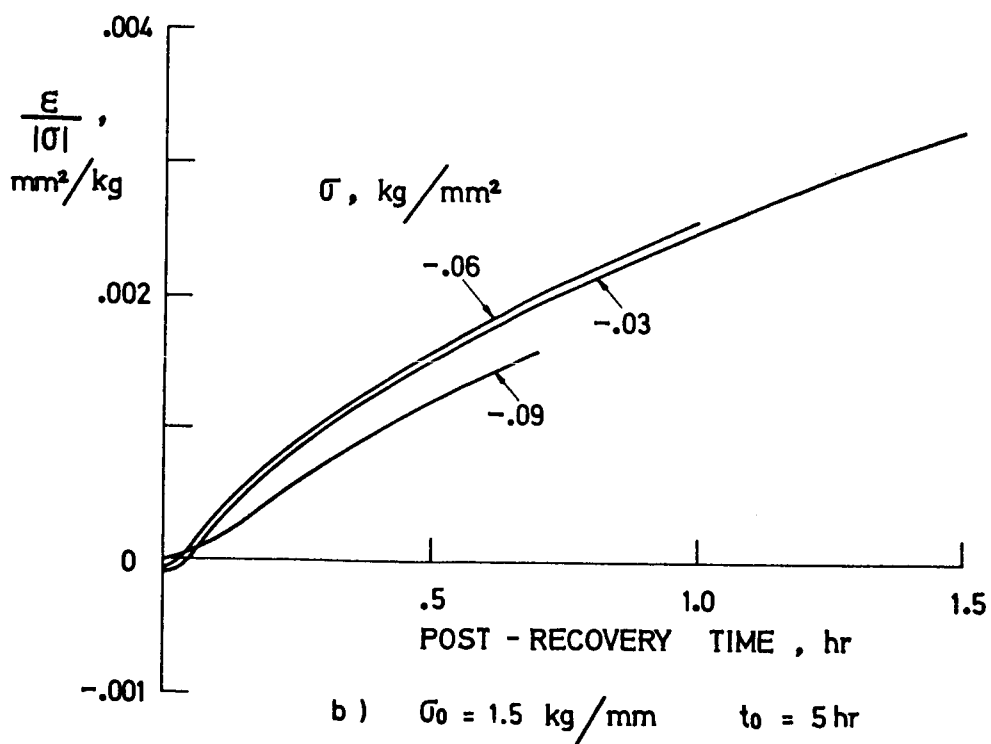
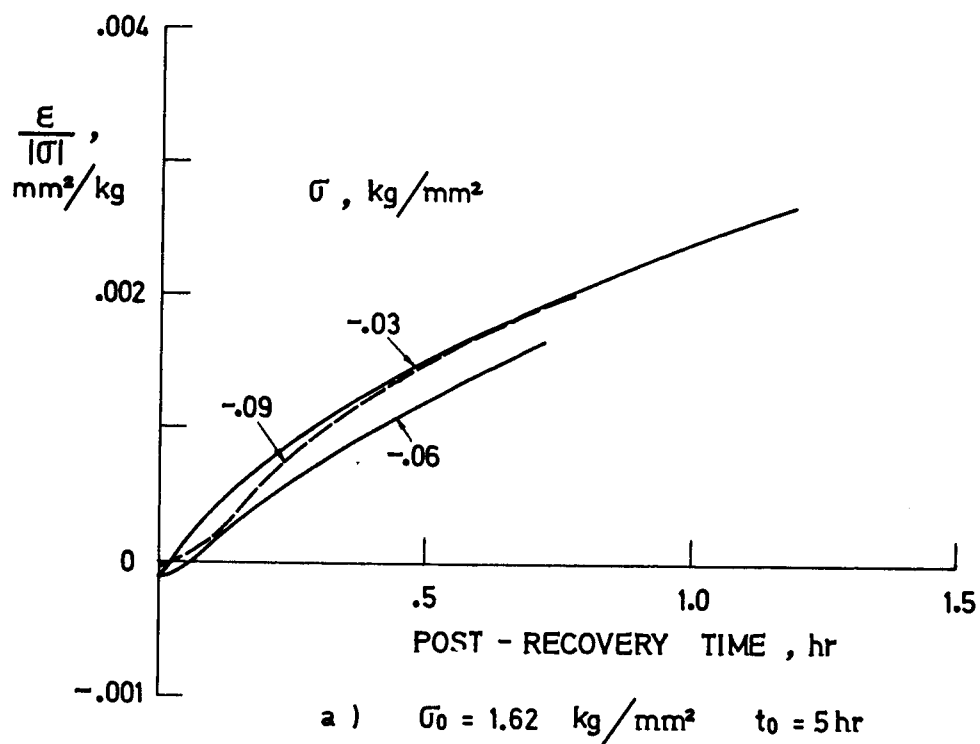


FIG. 14 CREEP OF COMMERCIALLY PURE ALUMINUM SHEET AT 200°C AFTER DECREMENTAL STRESS CHANGE, EXCLUDING RECOVERY TIME

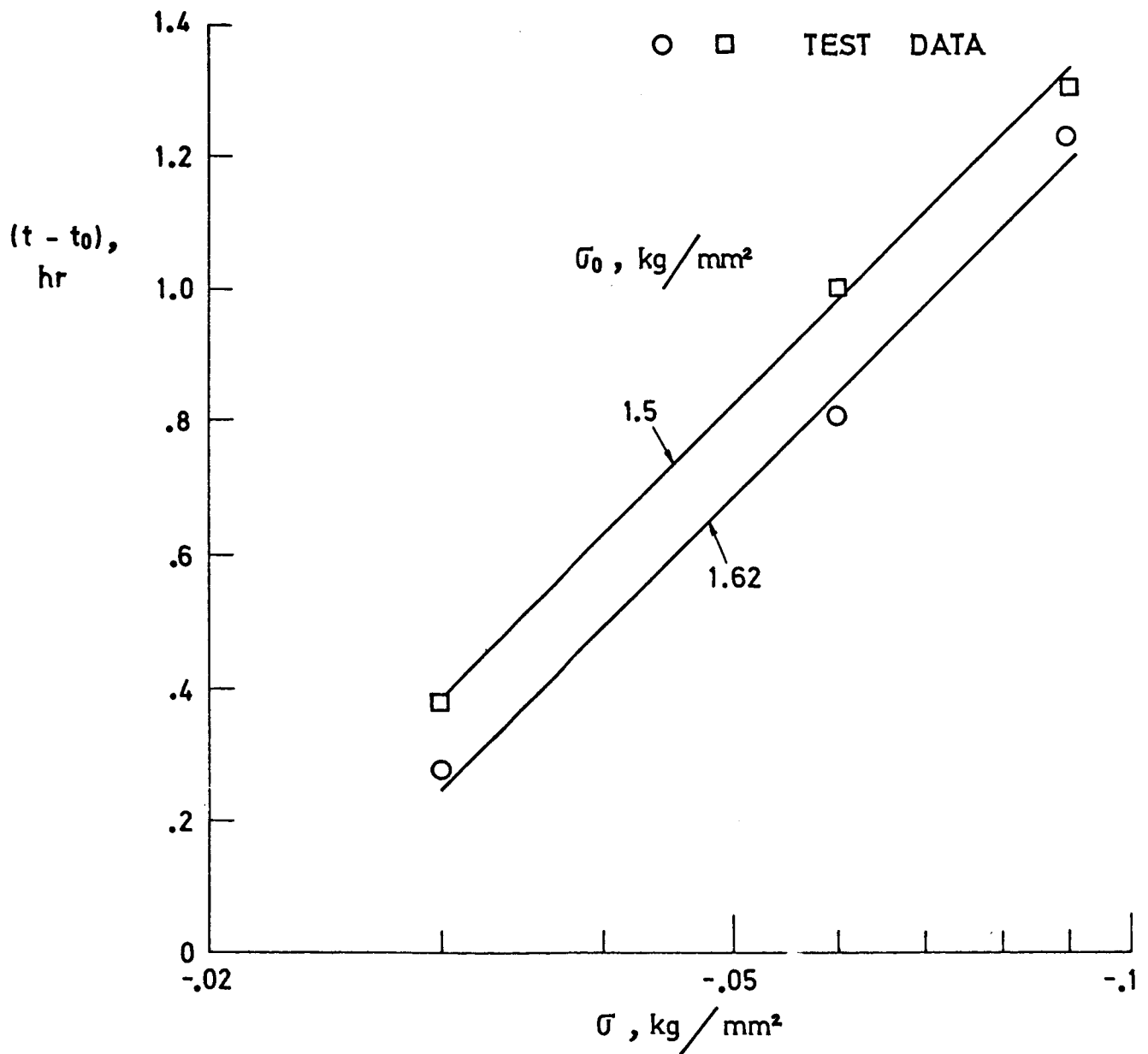


FIG. 15 RELATION BETWEEN DECREMENT OF STRESS AND RECOVERY TIME FOR COMMERCIALLY PURE ALUMINUM SHEET AT 200°C, $t_0 = 5$ hr